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# Optical bistability in an electron–hole system: the effect of phonons

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**Abstract.** We consider an electron–hole system illuminated by a laser and show that the system shows bistable and multistable behaviours. The mechanisms responsible for these behaviours are discussed in terms of excitonic and phonon interactions. It is found that both the excitonic and the phonon effects contribute to the multistability in the system. With an increase in the strength of the phonon interactions, the system passes from a phase of multistability to a phase of bistability; the reverse happens as the strength of the electron–hole interaction increases.

## 1. Introduction

Optical bistability forms an important aspect of the study of non-linear optical interactions in solids. Ever since its experimental demonstration in semiconductors by Gibbs *et al* (1976, 1979), Venkatesan and McCall (1977) and Miller and Smith (1979), there has been a tremendous rise in its study, which is important from both physical and technological points of view. While the physics of non-linear optical interactions derives its richness from its dependence on many-body effects in solids (Lowenau *et al* 1982, Haug and Schmitt-Rink 1984, Chemla *et al* 1988), the study of optical bistability is important technologically for its possible applications in constructing optical memory elements.

The problem of optical bistability in semiconductors has been studied by considering the rate equations for electron–hole pair amplitude, the light field amplitude and the density of electrons and holes (Goll and Haken 1980, 1983, Shrivastava and Tripathi 1984, Misra *et al* 1986). On the other hand, Lowenau *et al* (1982) studied the phenomenon by calculating the complex, non-linear dielectric function for arbitrary free-carrier concentrations from an integral equation for the polarization function.

Recently, there have been some attempts to understand the mechanisms responsible for bistable and multistable behaviour in low-dimensional solids, particularly polymers (Li *et al* 1990a, b). These authors made their analyses by considering a polymer inside an optical cavity, treating the cavity field, excitons and phonon modes as damped oscillators and neglecting the quantum fluctuations. They have shown that multistability in the system results from phonon interactions. However, we have recently shown (Misra *et al* 1986) that multistability would result even without phonon interactions. In view of the findings of Li *et al* (1990a, b), we consider in this work a phonon-coupled electron–hole system illuminated by a laser. Unlike in the earlier works, where the interactions of photons and phonons are considered with excitons (Li *et al* 1990a, b), we consider the interaction of these modes with electrons and holes separately, thus avoiding the ambiguity of treating the excitons as composite bosons. The electron–hole interaction is considered in terms

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of fermion operators. Since the treatment of the problem does not involve any specific dimension, it should be valid for any arbitrary dimension.

The paper is organized in the following way. In section 2, we construct and discuss the Hamiltonian of a phonon-coupled electron–hole system illuminated by a laser. In section 3, the equations of motion are derived. Section 4 analyses the steady-state solutions, and in section 5, we discuss the mechanisms responsible for bistability and multistability in the system considered and present our results.

## 2. The Hamiltonian

An electron–hole system in the presence of phonon interactions is described by the Hamiltonian

$$\mathcal{H} = \sum_k \mathcal{E}_k^{\mathrm{e}} c_k^+ c_k + \sum_k \mathcal{E}_k^{\mathrm{h}} b_k^+ b_k + \sum_q \hbar\omega_q a_q^+ a_q + \mathrm{i} \sum_{k,q} D_q [a_q (c_{k+q}^+ c_k + b_{k+q}^+ b_k) - a_q^+ (c_{k-q}^+ c_k + b_{k-q}^+ b_k)] + J \sum_k c_k^+ b_{-k}^+ c_k b_{-k} \tag{2.1}$$

where $\mathcal{E}_k^{\mathrm{e}}$, $\mathcal{E}_k^{\mathrm{h}}$ and $\hbar\omega_q$ are the electron, hole and phonon energies; the electron–phonon and hole–phonon couplings are described by the function $D_q$; $J$ is the electron–hole coupling energy or, in other words, the energy of an exciton and $(c_k^+, c_k)$, $(b_k^+, b_k)$ and $(a_q^+, a_q)$ are the (creation, annihilation) operators for electrons, holes and phonons respectively.

We consider a sample that is described by the above Hamiltonian in an optical cavity and exposed to an external laser field described by the vector potential

$$A = (2\pi\hbar c^2/\Omega n^2)^{1/2} e(B + B^+) \tag{2.2}$$

where $e$ is the polarization vector, $\Omega$ is the photon frequency and $n$ is the index of refraction of the medium. Furthermore,

$$B = \alpha_0 + \beta_0 \mathrm{e}^{-\mathrm{i}\Omega t} \tag{2.3a}$$

$$B^+ = \alpha_0^+ + \beta_0^+ \mathrm{e}^{\mathrm{i}\Omega t} \tag{2.3b}$$

where $(\alpha_0^+, \alpha_0)$ and $(\beta_0^+, \beta_0)$ are the creation and annihilation operators for the stationary cavity and incident photons, respectively. The electron–hole pair creation is described by (Hanamura 1977, Misra *et al* 1986)

$$\mathcal{H}_{\mathrm{I}} = \sum_k G_k (c_k^+ b_{-k}^+ B + c_k b_{-k} B^+) \tag{2.4}$$

where $G_k$ is proportional to the optical dipole matrix element for band-to-band transitions, which also depends on the material parameters like the refractive index. Equation (2.4) includes only resonant terms, i.e. an electron–hole pair is created by absorption of a photon or an electron–hole pair recombines with emission of a photon (rotating-wave approximation). It should be noted that in equation (2.4) we have neglected the spatial variation of the cavity field in view of the fact that the photon wavevector is small on the electronic scale.

Describing the Hamiltonian of the free light field of the sample by $\mathcal{H}_{\text{light}}$, where

$$\mathcal{H}_{\text{light}} = \hbar\Omega\alpha_0^+\alpha_0 \tag{2.5}$$

we obtain the total Hamiltonian $\mathcal{H}_{\text{T}}$ from equations (2.1), (2.4) and (2.5) and write it as

$$\begin{aligned}\mathcal{H}_{\text{T}} = &\sum_k \mathcal{E}_k^{\text{e}} c_k^+ c_k + \sum_k \mathcal{E}_k^{\text{h}} b_k^+ b_k + \sum_q \hbar\omega_q a_q^+ a_q \\ &+ \hbar\Omega\alpha_0^+\alpha_0 + \mathrm{i}\sum_{k,q} D_q[a_q(c_{k+q}^+ c_k + b_{k+q}^+ b_k) - a_q^+(c_{k-q}^+ c_k + b_{k-q}^+ b_k)] \\ &+ J\sum_k c_k^+ b_{-k}^+ c_k b_{-k} + \sum_k G_k(c_k^+ b_{-k}^+ B + c_k b_{-k} B^+).\end{aligned} \tag{2.6}$$

Our Hamiltonian (2.6) is distinguished from that given in equation (1) of Li *et al* (1990b) in the sense that it is expressed entirely in terms of fermion and boson operators, and thus the equations of motion that we shall derive will be free from the ambiguity of using the excitons as bosons. Furthermore, since our main interest is to investigate the non-linearity in the interaction of the light field with the system of electrons and holes, we have neglected for simplicity the electron–electron and hole–hole interactions, which mainly renormalize the electron and hole energies.

## 3. Equations of motion

The equations of motion for the electron and hole number density, electron–hole pair amplitude and cavity photon and phonon operators can be written down, using equation (2.6) and Heisenberg's equation:

$$\mathrm{d}F/\mathrm{d}t = (\mathrm{i}/\hbar)[H_{\text{T}}, F]. \tag{3.1}$$

These are:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(c_k^+ c_k + b_k^+ b_k) = &-\frac{\mathrm{i}}{\hbar}[(G_k c_k^+ b_{-k}^+ + G_{-k} c_{-k}^+ b_k^+)B - (G_k c_k b_{-k} + G_{-k} c_{-k} b_k)B^+] \\ &+ \frac{1}{\hbar}\sum_q D_q[a_q(c_k^+ c_{k-q} - c_{k+q}^+ c_k) - a_q^+(c_k^+ c_{k+q} - c_{k-q}^+ c_k)] \\ &+ \frac{1}{\hbar}\sum_q D_q[a_q(b_k^+ b_{k-q} - b_{k+q}^+ b_k) - a_q^+(b_k^+ b_{k+q} - b_{k-q}^+ b_k)]\end{aligned} \tag{3.2}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(c_k^+ b_{-k}^+ + c_{-k}^+ b_k^+) = &-\frac{\mathrm{i}}{\hbar}\{[G_k(c_k^+ c_k + b_{-k}^+ b_{-k} - 1) \\ &+ G_k(c_{-k}^+ c_{-k} + b_k^+ b_k - 1)]B^+ - (\mathcal{E}_k^{\text{e}} + \mathcal{E}_k^{\text{h}})(c_{-k}^+ b_k^+ + c_k^+ b_{-k}^+)\} \\ &- \frac{1}{\hbar}\sum_q D_q[a_q(c_{k+q}^+ b_{-k}^+ + c_{-k+q}^+ b_k^+ + b_{k+q}^+ c_{-k}^+ + b_{-k+q}^+ c_k^+) \\ &- a_q^+(c_{k-q}^+ b_{-k}^+ + c_{-k+q}^+ b_k^+ + b_{k-q}^+ c_{-k}^+ + b_{k-q}^+ c_k^+)] \\ &- \frac{\mathrm{i}J}{\hbar}[(1 - c_k^+ c_k - b_{-k}^+ b_{-k})c_k^+ b_{-k}^+ + (1 - c_{-k}^+ c_{-k} - b_k^+ b_k)c_{-k}^+ b_k^+]\end{aligned} \tag{3.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_0 = \frac{-\mathrm{i}}{\hbar}\left(\hbar\Omega\alpha_0 + \sum_k G_k c_k b_{-k}\right) \tag{3.4}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}a_q = \frac{-\mathrm{i}}{\hbar}\sum_k D_q(c^+_{k-q}c_k + b^+_{k-q}b_k) - \mathrm{i}\omega_q a_q. \tag{3.5}$$

Defining

$$(c^+_k c_k + b^+_k b_k) = (c^+_{-k}c_{-k} + b^+_{-k}b_{-k}) = n_k \tag{3.6}$$

$$(c^+_k c_{k-q} + b^+_k b_{k-q}) = (c^+_{k+q}c_k + b^+_{k+q}b_k) = n_k(q) \tag{3.7}$$

$$(c_k b_{-k} + c_{-k}b_k) = A \tag{3.8}$$

$$(c_{k+q}b_{-k} + c_{-k+q}b_k) = (b_{k+q}c_{-k} + b_{-k+q}c_k) = A(q) \tag{3.9}$$

equations (3.2) to (3.5) can be written as

$$\mathrm{d}n_k/\mathrm{d}t = -(\mathrm{i}/\hbar)G_k(A^+B - AB^+) \tag{3.10}$$

$$\begin{aligned}\mathrm{d}A^+/\mathrm{d}t &= (-\mathrm{i}/\hbar)[2G_k(n_k-1)B^+ - (\mathcal{E}^e_k + \mathcal{E}^h_k)A^+] \\ &\quad - (1/\hbar)\sum_q 2D_q[a_q A^+(q) - a^+_q A^+(-q)] - (\mathrm{i}/\hbar)J(1-n_k)A^+\end{aligned} \tag{3.11}$$

$$\mathrm{d}\alpha_0/\mathrm{d}t = (-\mathrm{i}/\hbar)(\hbar\Omega\alpha_0 + \tfrac{1}{2}G_k A) \tag{3.12}$$

$$\mathrm{d}a_q/\mathrm{d}t = (-1/\hbar)D_q n_k(-q) - \mathrm{i}\omega_q a_q \tag{3.13}$$

where we have assumed $G_k = G_{-k}$. The summations over $k$ in equations (3.4) and (3.5) are removed because only a single electron is involved in the scattering with a photon and a phonon at a given instant. Introducing the time dependence of the light field and electron–hole pair amplitude operators in the following way:

$$\tilde{\alpha}_0 = \alpha_0\,\mathrm{e}^{\mathrm{i}\Omega t} \tag{3.14}$$

$$\tilde{B} = B\,\mathrm{e}^{\mathrm{i}\Omega t} \tag{3.15}$$

$$\tilde{A} = A\,\mathrm{e}^{\mathrm{i}\Omega t} \tag{3.16}$$

$$\tilde{A}(\pm q) = A(\pm q)\mathrm{e}^{\mathrm{i}\Omega t} \tag{3.17}$$

the rate equations (3.10)–(3.13) become

$$\mathrm{d}n_k/\mathrm{d}t = -(\mathrm{i}/\hbar)G_k(\tilde{A}^+\tilde{B} - \tilde{A}\tilde{B}^+) \tag{3.18}$$

$$\begin{aligned}\mathrm{d}\tilde{A}^+/\mathrm{d}t &= -(2\mathrm{i}/\hbar)G_k(n_k-1)\tilde{B}^+ + (\mathrm{i}/\hbar)(\mathcal{E}^e_k + \mathcal{E}^h_k)\tilde{A}^+ \\ &\quad - (2/\hbar)\sum_q D_q[a_q\tilde{A}^+(q) - a^+_q\tilde{A}^+(-q)] \\ &\quad - \mathrm{i}\Omega\tilde{A}^+ - (\mathrm{i}/\hbar)J(1-n_k)\tilde{A}^+\end{aligned} \tag{3.19}$$

$$\mathrm{d}\tilde{\alpha}_0/\mathrm{d}t = (-\mathrm{i}/2\hbar)G_k\tilde{A} \tag{3.20}$$

$$\mathrm{d}a_q/\mathrm{d}t = -(1/\hbar)D_q n_k(-q) - \mathrm{i}\omega_q a_q. \tag{3.21}$$

Since we are interested in possible optical bistability and multistability in the variations of output laser intensity as a function of input laser intensity, we follow a semiclassical approach. We then replace the operators by their mean values defined as classical variables. Thus equations (3.18) to (3.21) become

$$\mathrm{d}\langle n_k\rangle/\mathrm{d}t = -(1/\hbar)G_k\langle(\tilde{A}^+\tilde{B} - \tilde{A}\tilde{B}^+)\rangle + \gamma_1\langle n_k\rangle \tag{3.22}$$

$$\begin{aligned}\mathrm{d}\langle\tilde{A}^+\rangle/\mathrm{d}t = &-(2\mathrm{i}/\hbar)G_k\langle(n_k - 1)\tilde{B}\rangle + (\mathrm{i}/\hbar)(\mathcal{E}_k^{\mathrm{e}} + \mathcal{E}_k^{\mathrm{h}})\langle\tilde{A}^+\rangle \\ &- (2/\hbar)\sum_q D_q\langle a_q\tilde{A}^+(q) - a_q^+\tilde{A}^+(-q)\rangle - \mathrm{i}\Omega\langle\tilde{A}^+\rangle \\ &- (\mathrm{i}/\hbar)J\langle(1 - n_k)\tilde{A}^+\rangle + \gamma_2\langle\tilde{A}^+\rangle\end{aligned} \tag{3.23}$$

$$\mathrm{d}\langle\tilde{\alpha}_0\rangle/\mathrm{d}t = -(\mathrm{i}/2\hbar)G_k\langle\tilde{A}\rangle + \gamma_3\langle\tilde{\alpha}_0\rangle \tag{3.24}$$

$$\mathrm{d}\langle a_q\rangle/\mathrm{d}t = -(1/\hbar)D_q\langle n_k(-q)\rangle - \mathrm{i}\omega_q\langle a_q\rangle + \gamma_4\langle a_q\rangle \tag{3.25}$$

where the decay constants $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ are introduced phenomenologically. The damping terms result from incoherent processes. Since the damping mechanism is statistical in nature and is due to random processes, it has been included in the theory phenomenologically. While the phenomenological treatment is certainly not valid under all circumstances, it has nevertheless been found to represent in many cases the proper qualitative or even quantitative description of the observed phenomena. Furthermore, this phenomenological procedure can be justified from first principles by coupling the system to a heat bath (Haken 1970).

## 4. Steady-state analysis

Since our main interest is to investigate the non-linearity in the laser-illuminated electron–hole system, we neglect all quantum fluctuations and assume a mean-field decoupling

$$\langle PQ\rangle = \langle P\rangle\langle Q\rangle. \tag{4.1}$$

However, we shall show that our final results are independent of this decoupling. Furthermore, in a spatially homogeneous system, conservation of momentum holds (Bogoliubov 1971) and we set

$$\langle n_k(q)\rangle = \langle n_k(-q)\rangle = \langle n_k\rangle \tag{4.2}$$

$$\langle\tilde{A}(q)\rangle = \langle\tilde{A}(-q)\rangle = \langle\tilde{A}\rangle. \tag{4.3}$$

Using equations (4.1) to (4.3) in equations (3.22) to (3.25), we obtain under steady-state conditions

$$(-\mathrm{i}/\hbar)G_k(\langle\tilde{A}^+\rangle\langle\tilde{B}\rangle - \langle\tilde{A}\rangle\langle\tilde{B}^+\rangle) + \gamma_1\langle n_k\rangle = 0 \tag{4.4}$$

$$\begin{aligned}(\mathrm{i}/\hbar)(\mathcal{E}_k^{\mathrm{e}} + \mathcal{E}_k^{\mathrm{h}})\langle\tilde{A}^+\rangle &- (2\mathrm{i}/\hbar)G_k(\langle n_k\rangle - 1)\langle B^+\rangle - (\mathrm{i}/\hbar)J(1 - n_k)\langle\tilde{A}^+\rangle \\ &+ \gamma_2\langle\tilde{A}^+\rangle - \mathrm{i}\Omega\langle\tilde{A}^+\rangle - (2/\hbar)\sum_q(\langle a_q\rangle - \langle a_q^+\rangle)\langle\tilde{A}^+\rangle = 0\end{aligned} \tag{4.5}$$

$$-(\mathrm{i}/2\hbar)G_k\langle\tilde{A}\rangle + \gamma_3\langle\tilde{\alpha}_0\rangle = 0 \tag{4.6}$$

and

$$-(1/\hbar)D_q\langle n_k\rangle - \mathrm{i}\omega_q\langle a_q\rangle + \gamma_4\langle a_q\rangle = 0. \tag{4.7}$$

From equation (4.7) we obtain

$$\langle a_q\rangle = D_q\langle n_k\rangle/[\hbar(\gamma_4 - \mathrm{i}\omega_q)] \tag{4.8}$$

$$\langle a_q^+\rangle = D_q\langle n_k\rangle/[\hbar(\gamma_4 + \mathrm{i}\omega_q)] \tag{4.9}$$

and

$$\langle a_q\rangle - \langle a_q^+\rangle = 2\mathrm{i}\omega_q D_q\langle n_k\rangle/[\hbar(\gamma_4^2 + \omega_q^2)]. \tag{4.10}$$

Substituting equation (4.10) in equation (4.5) we have

$$\langle\tilde{A}^+\rangle = \frac{2G_k(1-\langle n_k\rangle)\langle\tilde{B}^+\rangle}{\hbar\Omega - (\mathcal{E}_k^{\mathrm{e}} + \mathcal{E}_k^{\mathrm{h}}) + J(1-\langle n_k\rangle) + g\langle n_k\rangle + \mathrm{i}\hbar\gamma_2} \tag{4.11}$$

where

$$g = (4/\hbar)\sum_q[D_q^2\omega_q/(\gamma_4^2 + \omega_q^2)]. \tag{4.12}$$

Substituting equation (4.12) and its complex conjugate in equation (4.4), we obtain, using equation (4.1),

$$\frac{4G_k^2}{\hbar^2\gamma_1\gamma_2}\langle\tilde{B}^+\tilde{B}\rangle = \frac{\langle n_k\rangle}{(1-\langle n_k\rangle)}\left[1 + \left(\frac{\hbar\Omega - \mathcal{E}_k^{\mathrm{e}} - \mathcal{E}_k^{\mathrm{h}}}{\hbar\gamma_2} + \frac{J}{\hbar\gamma_2}(1-\langle n_k\rangle) + \frac{g}{\hbar\gamma_2}\langle n_k\rangle\right)^2\right]. \tag{4.13}$$

Using equations (2.3*a*), (3.14), (3.15) and (4.6), we have

$$\langle\beta_0\rangle = \langle\tilde{B}\rangle - (\mathrm{i}G_k/2\hbar\gamma_3)\langle\tilde{A}\rangle. \tag{4.14}$$

From equations (4.11), (4.13) and (4.14) we obtain, using (4.1):

$$\frac{4G_k^2}{\hbar^2\gamma_1\gamma_2}\langle\beta_0^+\beta_0\rangle = \frac{\langle n_k\rangle}{(1-\langle n_k\rangle)}\left[\left(1 + \frac{G_k^2}{\hbar^2\gamma_2\gamma_3}(1-\langle n_k\rangle)\right)^2 + \left(\frac{\hbar\Omega - (\mathcal{E}_k^{\mathrm{e}} + \mathcal{E}_k^{\mathrm{h}})}{\hbar\gamma_2} + \frac{J}{\hbar\gamma_2}(1-\langle n_k\rangle) + \frac{g}{\hbar\gamma_2}\langle n_k\rangle\right)^2\right]. \tag{4.15}$$

Thus our final results are independent of the decoupling procedure used (equation (4.1)). We define the dimensionless quantities

$$(4G_k^2/\hbar^2\gamma_1\gamma_2)\langle B^+B\rangle = I_{\mathrm{o}} \tag{4.16}$$

$$(4G_k^2/\hbar^2\gamma_1\gamma_2)\langle\beta_0^+\beta_0\rangle = I_{\mathrm{i}} \tag{4.17}$$

$$c_1 = [\hbar\Omega - (\mathcal{E}_k^{\mathrm{e}} + \mathcal{E}_k^{\mathrm{h}})]/\hbar\gamma_2 \tag{4.18}$$

$$c_2 = G_k^2/\hbar^2\gamma_2\gamma_3 \tag{4.19}$$

$$d = J/\hbar\gamma_2 \tag{4.20}$$

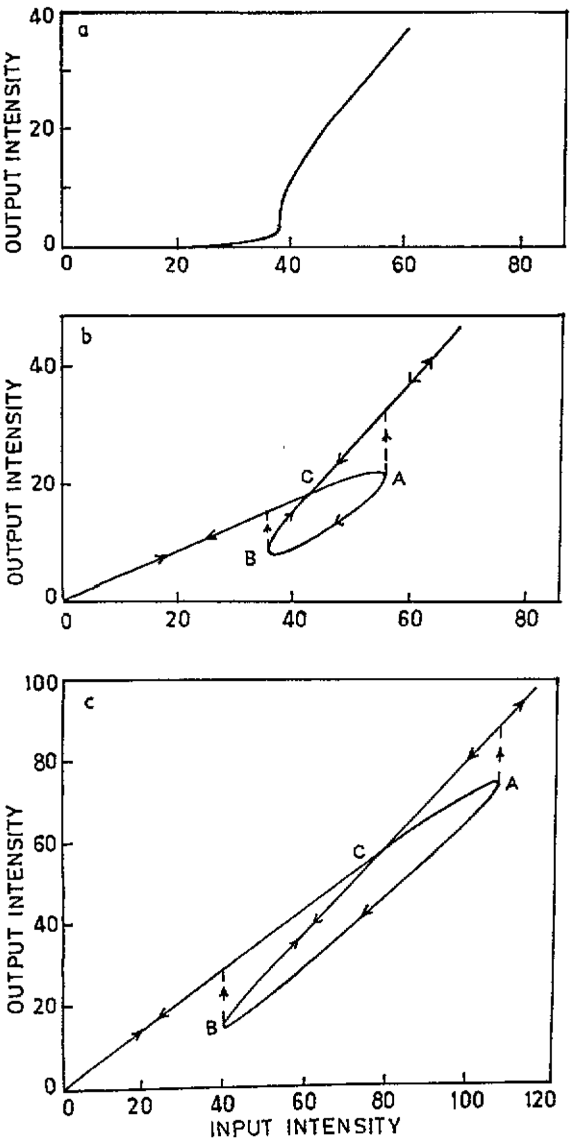


**Figure 1.** Output intensity versus input intensity for $c_1 = 0$, $c_2 = 10$, $p = 1$, and (*a*) $d = 0$, (*b*) $d = -10$ and (*c*) $d = -18$.

and

$$p = g/\hbar\gamma_2. \tag{4.21}$$

Equations (4.13) and (4.14) can be written with the help of equations (4.16)–(4.21) as

$$I_i = [\langle n_k\rangle/(1-\langle n_k\rangle)]\{[1+c_2(1-\langle n_k\rangle)]^2+[c_1+d(1-\langle n_k\rangle)+p\langle n_k\rangle]^2\} \tag{4.22}$$

and

$$I_o = [\langle n_k\rangle/(1-\langle n_k\rangle)]\{1+[c_1+d(1-\langle n_k\rangle)+p\langle n_k\rangle]^2\}. \tag{4.23}$$

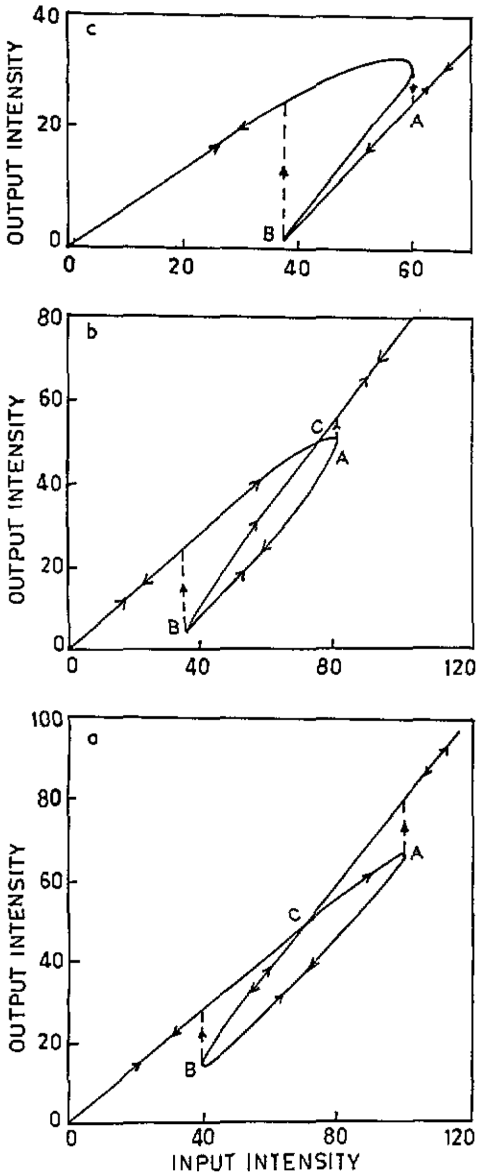


**Figure 2.** Output intensity versus input intensity for $c_1 = 1$, $c_2 = 10$, $d = -18$, and (*a*) $p = 0$, (*b*) $p = 3$ and (*c*) $p = 10$.

## 5. Results and discussion

In equations (4.22) and (4.23) $I_i$ and $I_o$ are input and output laser intensities, respectively. Both the equations are cubic as functions of $\langle n_k \rangle$. In the absence of electron–phonon, hole–phonon and electron–hole interactions, only equation (4.22) is a cubic equation, but equation (4.23) is not. Thus, in the absence of these interactions, the system can only show bistable behaviour and the condition is (Shrivastava and Tripathi 1984):

$$3c_2(2c_2 + 5 + 9c_1^2)(c_1^2 + 1) + 8c_1^2 < c_2^3(c_1^2 + 1) \tag{5.1}$$

which is obtained from $dI_i/d\langle n_k \rangle = 0$ for $p = 0$ and $d = 0$. For finite values of either $d$ or $p$ or both, both equations are cubic and contribute to the multistability. Indeed, in one of our earlier papers (Misra *et al* 1986), we have shown that the variation of output intensity as a function of input intensity shows multistable behaviour, even in the absence of phonon interactions. This clearly demonstrates that phonons alone are not indispensable for multistable behaviour.

Multistability results when each of $I_i$ and $I_o$ shows a maximum and a minimum as a function of $\langle n_k \rangle$, the conditions for which are obtained by equating to zero the first derivatives of $I_i$ and $I_o$ with respect to $\langle n_k \rangle$ and solving them for three real roots. Thus there would be multiple solutions if the following equations are satisfied simultaneously:

$$b_{i,o}^2/4 + a_{i,o}^3/27 < 0 \tag{5.2}$$

where i and o stand for equations (4.22) and (4.23);

$$a_i = (3a_1a_3 - a_2^2)/3a_1^2 \tag{5.3}$$

$$b_i = (2a_2^3 - 9a_1a_2a_3 + 27a_1^2a_4)/27a_1^3 \tag{5.4}$$

with

$$a_1 = 2(p^2 + c_2^2 + d^2 + 2pd) \tag{5.5}$$

$$a_2 = -(5d^2 + 5c_2^2 + 3p^2 + 8pd + 2c_1p + 2c_1d + 2c_2) \tag{5.6}$$

$$a_3 = 4(d^2 + c_2^2 + c_1d + c_1p + dp + c_2) \tag{5.7}$$

$$a_4 = -(c_1^2 + c_2^2 + d^2 + 2c_1d + 2c_2 + 1) \tag{5.8}$$

and $a_o$ and $b_o$ are $a_i$ and $b_i$ with $c_2 = 0$.

In figures 1 and 2 we have plotted the output intensity versus input intensity for different conditions. In figure 1 we have plotted $I_o$ versus $I_i$ for fixed values of $c_1$, $c_2$ and $p$, and varying $d$, i.e. by varying the strength of electron–hole interaction. It is found that, as the electron–hole strength is increased, the system shows pronounced multistable behaviour. However, at $d = 0$ (figure 1(*a*)) the system does not show multistability. In other words, excitonic effects are important for multistability. In figure 2(*a*) we have plotted the output intensity as a function of input intensity for fixed values of $c_1$, $c_2$ and $d$. It is found that the system shows multistable behaviour even for $p = 0$ (figure 2(*a*)), which indicates that phonons are not indispensable for multistability. For small changes in phonon strength, i.e. for finite but small positive values of the phonon energy, the system shows multistable behaviour (figure 2(*b*)), but the turning points, A and B, show up at different input intensities. To illustrate this point, we consider the curve for $p = 0$ (figure 2(*a*)). As the input laser intensity increases, the output intensity increases and reaches the point A. Since A is unstable, $I_o$ will either go up with increase in the input intensity or come down along the curve AB with decrease in the input intensity. Similarly at B, the output intensity would either go up and then decrease or pass along the curve BC and increase. Thus, the system shows multistable behaviour. However, as the strength of the phonon interaction increases, the system passes through a stage of bistability (figure 2(*c*)), i.e. with increase in $I_i$, $I_o$ passes through the first branch, then comes down at A and then increases. But with decrease in $I_i$, $I_o$ passes through the lower branch, comes down to B and then goes up and then decreases.

Before we conclude, we would like to compare our results with that of Li *et al* (1990a, b), who have also shown multistability in a system of excitons coupled to phonons and a light field. They have observed that virtual excitons, phonon modes and feedback of the cavity are all indispensable for such behaviour. Since they have treated excitons as bosons, their multistability results from a laser-illuminated boson system. On the other hand, we considered a coupled fermion–boson system illuminated by a laser. Thus our approach is free from the ambiguity of treating the excitons as composite bosons. While there is an overall agreement in the mechanisms responsible for multistability, the difference arises in the role of phonons. We have shown that phonons do contribute to the multistability but these are not indispensable, as observed by Li *et al* (1990a, b). This disagreement is due presumably to the difference in the systems considered.

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